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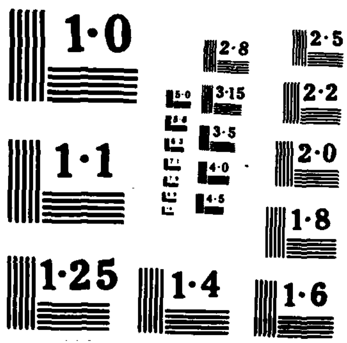
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Vector Analysis and Satellite Footprints

R. H. OTT
Electronics Research Laboratory
Laboratory Operations
The Aerospace Corporation
El Segundo, CA 90245

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Los Angeles, CA 90009-2960

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Wesley R. Dotts

WESLEY R. DOTTS, Lt, USAF
MOIE Project Officer
SD/CGX

Joseph Hess

JOSEPH HESS, GM-15
Director, AFSTC West Coast Office
AFSTC/WCO OL-AB

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The use of vector analysis for deriving algorithms for generating earth coverage footprints from satellites in arbitrary orbits is presented. Extensive analysis and results are given for two examples: 1) a satellite in geosynchronous orbit and 2) a satellite in subsynchronous orbit. The antenna beam on the satellite can be elliptic in cross section with arbitrary orientation of semi-minor and semi-major axes.

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CONTENTS

FIGURES

I. INTRODUCTION

The projections of antenna pattern characteristics into contours of constant power flux on the earth are called footprints. The coverage characteristics of these footprints are used in studying satellite communication links, to infer the (roll, pitch and yaw) variations in coverage as satellites depart from their nominal attitude, and in remote sensing applications to determine the ability to resolve surface features. The footprints are not only a function of the antenna pattern but also depend in a complicated way on the relationships or mapping between a coordinate system fixed to the antenna and the earth's coordinate system. The coordinate system on the antenna may also be different from the coordinate system used to describe the location of the satellite itself.

Since the kinetic energy of any satellite by definition is insufficient to escape the gravitational field of the primary body, its unperturbed orbit is periodic and closed. The period of revolution determined by Kepler's third law in the reference frame is

$$T^2 = \frac{2\pi}{\mu} a^3 \tag{1}$$

where a is the semi-major axis of the elliptic orbit and μ equals the product of the mass of the primary body and the universal gravitation constant, G. For the earth, a currently accepted value is

$$\mu = 3.98600800... \times 10^{14} \ m^3/sec^2. \tag{2}$$

A synchronous satellite has a period, T, equal to the sidereal period of rotation of its primary body; i.e., the time during which the earth makes a complete rotation relative to the vernal equinox. Note that the period for the earth is not 24 hours, because in one day, the earth rotates around its polar axis and also completes 1/365.24 of the annual earth orbit around the sun. Consequently, a geosynchronous satellite has a period given by

$$T_p = \left(1 - \frac{1}{365.242}\right) \times 24 \text{ hr.}$$

$$= 86164.09054 \text{ ephemeris seconds} \quad (3)$$

$$\cong 23^h 56^m\ 4^s \quad \text{mean solar day}$$

If

$$T = \alpha\, T_p \quad (4)$$

where $\alpha < 1$ the satellite is called subsynchronous.

In addition to the shape of the satellite antenna beam, the $1/R^2$ factor for power density dependence can modify the shape of a footprint if the latter corresponds to a constant power contour. However, Figure 1 shows that the most this variation can be is about 1.3 dB for the synchronous altitude. That is,

$$20 \log_{10}(R_{max}/R_{min}) \cong 1.3 \text{ dB.}$$

At subsynchronous altitudes, the variation in range becomes more pronounced.

Another possible modification of the footprint is caused by bending of the rays as they pass through the atmosphere. The effect would be greatest for stations on the horizon; i.e., stations which are required to operate at low elevation angles. For example, consider Anchorage, Alaska, at about 60°N latitude. Figure 2 shows the geometry for estimating the amount of bending of the rays. Thayer (1961) gives the following formula for the amount of bending of a radio ray in radians as it passes through an exponential atmosphere:

$$\tau(\text{rad.}) = 0.0003 \cot\sigma_o, \quad \sigma_o > 15° \quad (5)$$

where σ_o is the take-off angle (electron angle), which, from the geometry in Figure 2, is about 22°. This value for the take-off angle gives a total bending of about 7.4×10^{-4} radians or about 0.043 degrees. The take-off

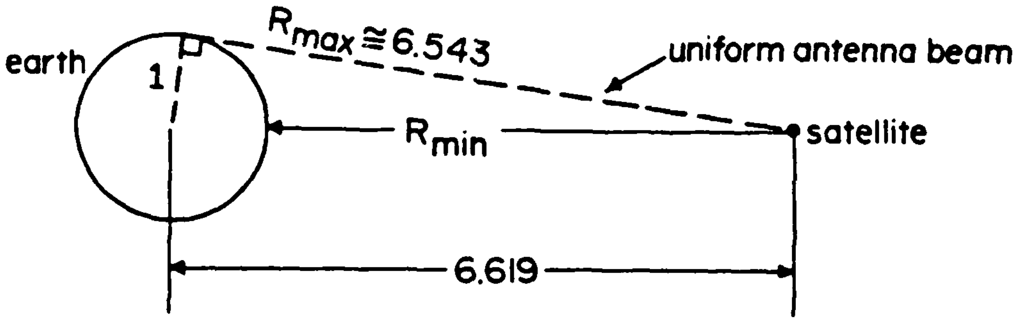

Figure 1. The effect of R^{-2} power density dependence on footprint shape. All distances are in earth radii.

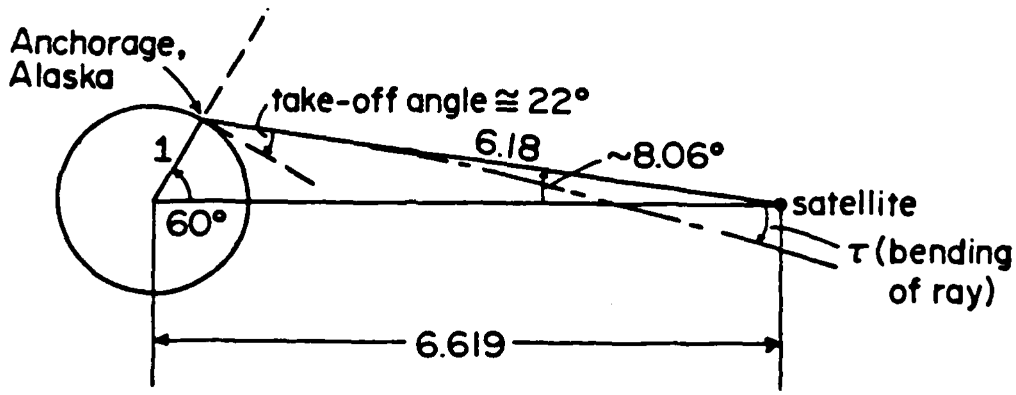

Figure 2. Geometry for estimating the amount of bending of a radio ray as it passes through an exponential atmosphere. All distances are in earth radii.

angle should be greater than about 5° in a satellite-communications link to avoid excessive bending of the rays.

A third modification of the antenna footprint is due to the oblate spheroidal shape of the earth. Again, this effect is on the order of a few milliradians. A fourth modification is the effect of atmospheric absorption which becomes significant at EHF frequencies. For example, at 10 GHz the atmospheric attenuation will be about 2 dB at the horizon (Bean and Dutton, 1966). Jacobs and Stacy (1971) develop expressions for computing the footprints of satellite antennas with circularly symmetric patterns which takes into account not only the R^{-2} factor but also atmospheric attenuation.

The footprint defines the area on the surface of the earth inside of which the system has a specified or greater level of sensitivity. The mathematical problem of finding the locus of the footprints can be solved in a straightforward manner using vector analysis. This report documents a computer program written to draw the locus of the footprints on computer generated maps. A particular feature of this analysis is the capability to generate footprints for a satellite which uses repetitively scanned beams to achieve the total required coverage.

II. VECTOR ANALYSIS

A. SUN-SYNCHRONOUS ORBIT, ANTENNA SCANNING WITH RESPECT TO BODY-FIXED, VEHICLE-CENTERED COORDINATE SYSTEM

In this section, the equations for the locus of intersection points, i.e., the footprint of the satellite antenna beam and the spherical earth, are derived. The particular orbit investigated is a "sun-synchronous" orbit for which the satellite orbit will pass over a given latitude at the same local time-of-day for each pass. Because of this similarity of lighting and the near-consistency of the local time-of-day, most meteorological satellites like the Defense Meteorological Satellite Program (Hollinger and Lo, 1984) are in this type of orbit. The inclination of the orbit with respect to the equator in the latitude-longitude coordinate system is shown in Figure 3. In the example considered in this report, $i \cong 98.1°$ and the circular altitude above the equator is about 833 km (Denner, 1982). The orbital period is about 101.5 minutes and the circular orbital velocity is about 6.58 km/sec, and the "grazing" orbit velocity is about 7.406 km/sec.

The geometry of the problem is displayed in Figure 3. In the derivation, the earth has unit radius so all distances are in earth radii. The position of the satellite in subsynchronous orbit from Figure 3 is given by the vector $\underline{S}$,

$$\underline{S} = s\,(\underline{e}_x \sin\theta\cos\phi - \underline{e}_y \sin\theta\sin\phi + \underline{e}_z \cos\theta) \tag{7}$$

where unit vectors are denoted by $\underline{e}$ throughout this report and with s determined from the Vis-viva ("living force", Escobal, 1965) equation for a circular orbit

$$v^2 = \mu/s \tag{8}$$

with $\mu = 1.000952348$, and v the circular velocity, and s in earth radii units (1 circular satellite unit $\cong$ 7.905293 km/sec.). From Figure 3,

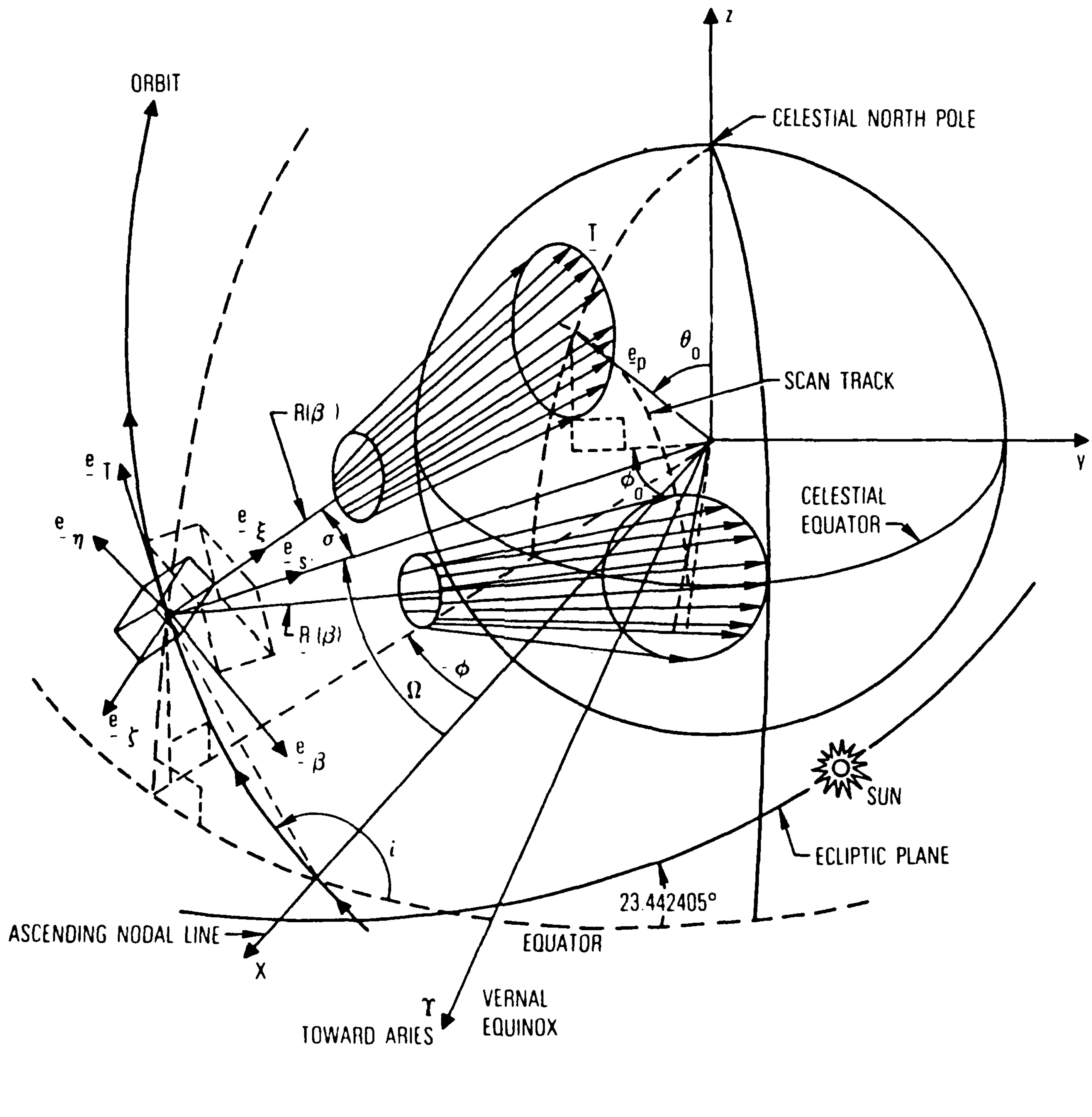

Figure 3. Geometry for footprints generated by a scanning antenna on a satellite in sub-synchronous orbit.

$$\underline{e}_x \cdot \underline{e}_s = \cos\Omega = \sin\theta\cos\phi \tag{9}$$

and

$$(\underline{e}_s \times \underline{e}_x) \cdot \underline{e}_y = \sin\Omega\sin i = \cos\theta \tag{10}$$

and

$$(\underline{e}_s \times \underline{e}_x) \cdot -\underline{e}_z = \sin\Omega\cos i = \sin\theta\sin\phi \tag{11}$$

Basically, s, i and time, t, in Figure 3 determine the orbit with the angular variable, Ω, given by

$$\Omega = \omega t = vt/s = \sqrt{\mu}\ t/s^{3/2} \tag{12}$$

From Figure 3, a vector from the satellite to an arbitrary point on the scan track is

$$\underline{R} = \underline{e}_p - \underline{S} \tag{13}$$

with the latitude $\left(\frac{\pi}{2} - \theta_o\right)$ and longitude, ϕ_o, of a point on the earth found from

$$\underline{e}_p = \underline{e}_x \sin\theta_o\cos\theta_o + \underline{e}_y\sin\theta_o\sin\phi_o + \underline{e}_z\cos\theta_o \tag{14}$$

The vector $\underline{R}$, in terms of a coordinate system centered on the satellite with the unit vector $\underline{e}_T$ pointing in the direction of motion and $\underline{e}_\beta$ chosen to form a triad, is

$$\underline{R} = D\left[-\cos\sigma\underline{e}_s + \sin\sigma(\sin\beta\underline{e}_\beta + \cos\beta\ \underline{e}_T)\right] \tag{15}$$

with $\beta = 2\pi$ radians/T, where T is the period to make a scan, and σ the angle between the boresight direction of the antenna and a radial vector from the center of the earth and the satellite. From Equations (14) and (15) and the fact that $\underline{e}_p \cdot \underline{e}_p = 1$ we find

$$1 = D^2 \left[\cos^2\sigma + \sin^2\sigma(\sin^2\beta + \cos^2\beta)\right] + 2s\, D\, \underline{R} \cdot \underline{e}_s + s^2$$

$$0 = D^2 - 2sD \cos\sigma + s^2 - 1 \qquad (16)$$

and solving for D

$$D = s \cos\sigma - \left[1 - s^2\sin^2\sigma\right]^{1/2} \qquad (17)$$

the negative root selected corresponding to the point on the earth closest to the satellite or on the "lit" side. For example, if $\sigma = \pi/4$,

$$D = \frac{s - \sqrt{2-s^2}}{\sqrt{2}} \qquad (18)$$

and if s = 1.131463449 e.r.

$$D \cong 0.200 \qquad (19)$$

and from Figure 4

$$D^2 = 1 + s^2 - 2s \cos\theta$$

or

$$\cos\theta = \frac{s^2 + 1 - D^2}{2s} = 0.989957$$

$$\theta \cong 8.127° \qquad (20)$$

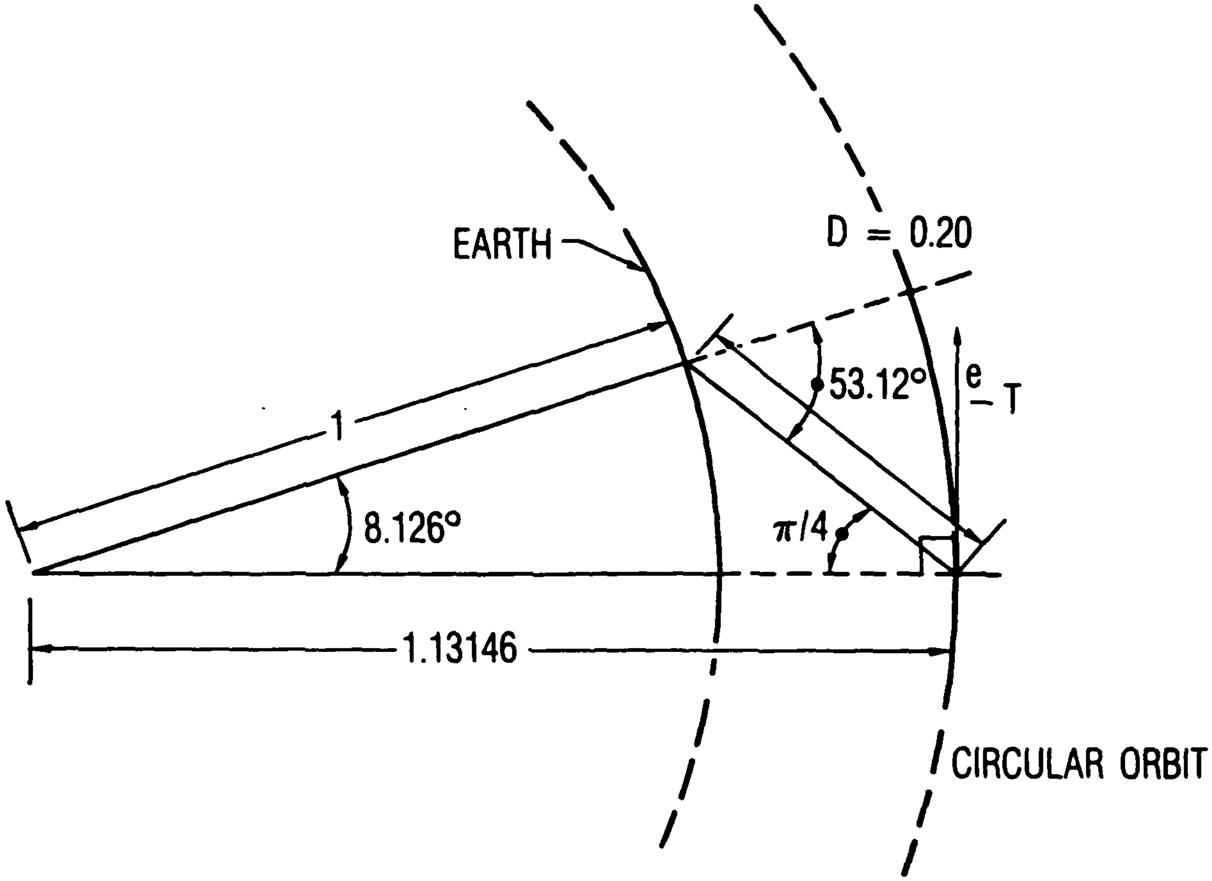

Figure 4. Parameters for a typical DMSP orbit.

To find the unit vector, $\underline{e}_T$, in the direction of motion, rotate frame Oxyz about the x-axis by the angle as

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin i & \cos i \\ 0 & -\cos i & \sin i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{21}$$

and the unit tangent vector, $\underline{e}_T$, in the xyz frame is

$$\underline{e}_T = - \underline{e}_x \sin\Omega + \underline{e}_z \cos\Omega \tag{22}$$

Therefore $\underline{e}_T$ in frame x'y'z' is from Equations (21) and (22)

$$\underline{e}_T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin i & \cos i \\ 0 & -\cos i & \sin i \end{bmatrix} \begin{bmatrix} -\underline{e}_x \sin\Omega \\ 0 \\ \underline{e}_z \cos\Omega \end{bmatrix} \tag{23}$$

$$\underline{e}_T = - \underline{e}_x \sin\Omega + \underline{e}_y \cos\Omega \cos i + \underline{e}_z \cos\Omega \sin i$$

Then, the third vector in the triad, $\underline{e}_s$, $\underline{e}_T$, $\underline{e}_\beta$ is defined by

$$\underline{e}_\beta = \underline{e}_T \times \underline{e}_s \tag{24}$$

and substituting Equations (7) and (23) into (24) gives

$$\underline{e}_\beta = \underline{e}_y \sin i - \underline{e}_z \cos i \tag{25}$$

A check on the unit vectors $\underline{e}_T$ and $\underline{e}_\beta$ is provided by Equations (13) and (16) which implies

$$|\underline{R} + \underline{S}|^2 = D^2 + s^2 - 2Ds \cos\sigma \tag{26}$$

Substituting for $\underline{R}$ from Equation (15) and from (23) and (25), in terms of $\underline{e}_x$, $\underline{e}_y$ and $\underline{e}_z$ shows indeed Equation (26) is satisfied.

Therefore, we can find typical colatitudes, θ_o, and longitudes, ϕ_o, along the scan track from

$$\underline{e}_p = \frac{\underline{S} + \underline{R}}{\sqrt{D^2 + s^2 - 2Ds\cos\sigma}} \tag{27}$$

and

$$\cos\theta_o = \underline{e}_p \cdot \underline{e}_z$$

$$\sin\theta_o \sin\phi_o = \underline{e}_p \cdot \underline{e}_y$$

$$\sin\theta_o \cos\phi_o = \underline{e}_p \cdot \underline{e}_x \tag{28}$$

Knowing θ_o and ϕ_o we can now construct a rectangular frame for defining the footprint; i.e., $\underline{e}_\xi$, $\underline{e}_\eta$ and $\underline{e}_\zeta$ in Figure 3. The first unit vector $\underline{e}_\xi$ is defined as

$$\underline{e}_\xi = \underline{R}/|\underline{R}| \tag{29}$$

and from Equations (7), (15), (23) and (25) and a great deal of algebra one finds

$$\begin{aligned}\underline{e}_\xi = -\underline{e}_x(\cos\sigma \cos\Omega + \sin\sigma \sin\Omega \cos\beta) + \underline{e}_y\,(-\cos\sigma \sin\Omega \cos i + \sin\sigma \\ \sin\beta \sin i + \sin\sigma \cos\beta \cos\Omega \cos i) \\ + \underline{e}_z(-\cos\sigma \sin\Omega \sin i - \sin\sigma \sin\beta \cos i + \sin\sigma \cos\beta \cos\Omega \sin i)\end{aligned} \tag{30}$$

From Figure 3

$$\underline{e}_\eta = \underline{e}_\xi \cos\sigma + \underline{e}_s \sin\sigma \tag{31}$$

$$= \frac{1}{\tan\sigma} \Big\{-\underline{e}_x\Big(\cos\sigma \cos\Omega + \sin\sigma \sin\Omega \cos\beta - \frac{\cos\Omega}{\cos\sigma}\Big) + \underline{e}_y\Big(-\cos\sigma \sin\Omega \cos i$$

$$+ \sin\sigma \sin\beta \sin i + \sin\sigma \cos\beta \cos\Omega \cos i + \frac{\sin\Omega \cos i}{\cos\sigma}\Big)$$

$$+ \underline{e}_z\Big(-\cos\sigma \sin\Omega \sin i - \sin\sigma \sin\beta \cos i + \sin\sigma \cos\beta \cos\Omega \sin i + \frac{\sin\Omega \sin i}{\cos\sigma}\Big)\Big\}$$

and the third vector in the triad $\underline{e}_\xi$, $\underline{e}_\eta$, $\underline{e}_s$ is

$$\underline{e}_\zeta = \underline{e}_\xi \times \underline{e}_\eta \tag{32}$$

$$= \underline{e}_x \sin\beta \sin\Omega + \underline{e}_y(\cos\beta \sin i - \sin\beta \cos i \cos\Omega)$$

$$- \underline{e}_z(\cos\beta \cos i + \sin\beta \sin i \cos\Omega)$$

A typical generator of the antenna footprint is

$$\underline{T} = t(w)\, \big[\underline{e}_\xi + a\big(\underline{e}_\eta \cos\omega + \underline{e}_\zeta \sin\omega\big)\big] \tag{33}$$

where a is the half-power antenna beam width. At a typical intersection point

$$\underline{S} + \underline{T} = \underline{e}_p \tag{34}$$

or

$$\underline{e}_p \cdot \underline{e}_p = 1 = s^2 + t^2 + 2\, \underline{S} \cdot \underline{T}$$

which becomes after a great deal of algebra

$$t^2 + \frac{2st(\cos\sigma + a\ \cos\omega\sin\sigma)}{1 + a^2} + \frac{(s^2-1)}{(1 + a^2)} = 0 \tag{35}$$

The solution for t closest to the satellite is

$$t(\omega) = \frac{1}{(1 + a^2)} \Big\{-s\ (-\cos\sigma + a\cos\omega\ \sin\sigma)$$

$$-\left[s^2(-\cos\sigma + a\cos\omega\ \sin\sigma)^2 - (s^2 - 1)(1 + a^2)\right]^{1/2}\Big\} \tag{36}$$

The colatitude, θ, and longitude, ϕ, of a typical intersection point are

$$\begin{aligned} \cos\theta &= \underline{e}_p \cdot \underline{e}_z \\ \sin\theta\ \cos\phi &= \underline{e}_p \cdot \underline{e}_x \\ \sin\theta\ \sin\phi &= \underline{e}_p \cdot \underline{e}_y \end{aligned} \tag{37}$$

and this completes the derivation for the subsynchronous case.

B. GEOSTATIONARY ORBIT

In this section the equations for the locus of intersection points of the satellite in geostationary orbit and a spherical earth are derived. The satellite is located in the equatorial plane (x-y plane in Figure 5) at an arbitrary longitude, λ_o, and at an arbitrary distance, s, from the origin. In vector notation, the satellite position is

$$\underline{S} = s(\underline{e}_x\ \cos\lambda_o + \underline{e}_y\ \sin\lambda_o). \tag{38}$$

The maximum of the antenna spot beam points to an arbitrary longitude and latitude (λ, ϕ), designated the aim point. In vector notation, the aim point is

$$\underline{e}_p = \underline{e}_x \cos\lambda\cos\phi + \underline{e}_y \sin\lambda\cos\phi + \underline{e}_z \sin\phi, \tag{39}$$

which is a unit vector since the sphere has unit radius.

In order to relate the shape of the antenna beam on the satellite to the locus of intersection points on the sphere, an orthogonal coordinate system $(\underline{e}_\xi, \underline{e}_\eta, \underline{e}_\zeta)$ is erected at the satellite. From Figure 5, a unit vector from the satellite to the aim point is given by:

$$\underline{e}_\xi = \frac{\underline{e}_p - \underline{S}}{|\underline{e}_p - \underline{S}|} . \tag{40}$$

Substituting Equations (38) and (39) into (40) gives

$$\underline{e}_\xi = [\underline{e}_x(\cos\lambda\ \cos\phi - s\ \cos\lambda_o) + \underline{e}_y(\sin\lambda\ \cos\phi - s\ \sin\lambda_o) + \underline{e}_z\ \sin\phi]/d_1 \tag{41}$$

with

$$d_1 = \left[1 + s^2 - 2s\ \cos\phi\ \cos(\lambda - \lambda_o)\right]^{1/2} . \tag{42}$$

When the aim point and subsatellite point (N in Figure 5) coalesce so that $\phi = 0$ and $\lambda = \lambda_o$,

$$\underline{e}_\xi(\phi = 0,\ \lambda = \lambda_o) = \underline{e}_x \cos\lambda_o - \underline{e}_y \sin\lambda_o \tag{43}$$

since

$$d_1 = s - 1 \tag{44}$$

in this case. A second unit vector in the right-handed system used at the satellite is $\underline{e}_\eta$. Let $\underline{e}_\eta$ lie in the ξ-z plane.

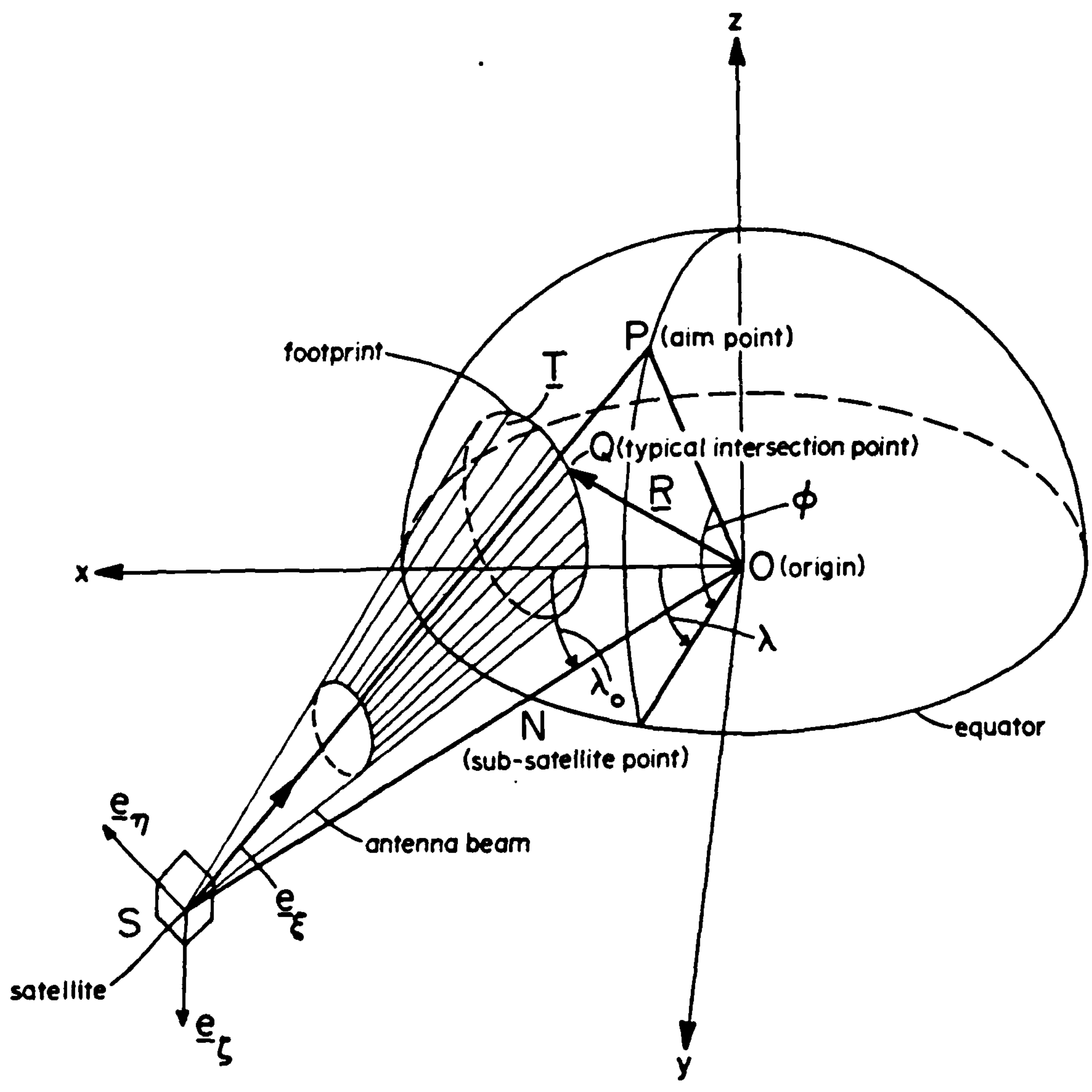

Figure 5. Geometry for the derivation of footprints. The geostationary satellite at S is in the x-y (equatorial) plane.

Where

$$\underline{e}_\eta = \frac{-\underline{e}_\xi + \alpha\underline{e}_z}{|-\underline{e}_\xi + \alpha\underline{e}_z|}, \tag{45}$$

and α is determined by the orthogonality condition

$$\underline{e}_\eta \cdot \underline{e}_\xi = 0. \tag{46}$$

Substituting Equations (41) and (45) into (46) yields

$$\alpha = d_1/\sin\phi, \tag{47}$$

$$d_2 = \sqrt{d_1{}^2 - \sin^2\phi}. \tag{48}$$

Using Equation (40), $\underline{e}_\eta$ can be expressed in terms of $\underline{e}_x$, $\underline{e}_y$, and $\underline{e}_z$ as follows:

$$\begin{aligned}\underline{e}_\eta = &-\underline{e}_x\sin\phi(\cos\lambda\ \cos\phi - s\ \cos\lambda_o)/d_1d_2\\ &-\underline{e}_y\sin\phi(\sin\lambda\ \cos\phi - s\ \sin\lambda_o)/d_1d_2\\ &+\underline{e}_z(d_2/d_1).\end{aligned} \tag{49}$$

In the case $\phi = 0$,

$$\underline{e}_\eta = \underline{e}_z\ (\phi = 0),$$

as of course is expected.

The third member of the triad is constructed to form a right-handed system; i.e.,

$$\underline{e}_\zeta = \underline{e}_\xi \times \underline{e}_\eta. \tag{50}$$

Substituting Equations (49) and (41) into (50) gives

$$\underline{e}_{\zeta} = \underline{e}_x(\sin\lambda\cos\phi - s\ \sin\lambda_o)/d_2$$

$$-\underline{e}_y(\cos\lambda\cos\phi - s\ \cos\lambda_o)/d_2. \tag{51}$$

Having taken care in deriving the orthogonal coordinate system at the satellite will permit writing an expression for a typical generator in the beam. From Figures 5 and 6, we can write

$$\underline{T} = t(\omega)[\underline{e}_{\xi} + \underline{e}_{\eta}(a\ \cos\omega\ \cos\beta - b\ \sin\omega\ \sin\beta)$$

$$+ \underline{e}_{\zeta}(a\ \cos\omega\ \sin\beta + b\ \sin\omega\ \cos\beta)], \tag{52}$$

where $t(\omega)$ is the length of the generator of the elliptic cone, a and b are the semi-minor and semi-major axes of the cross section of the elliptic cone, respectively, β is the tilt of the semi-major axis of the beam with respect to the η-ζ coordinate system, and ω is the parameter that generates the elliptic cone as ω varies from 0 to 2π radians. An elliptic beam can be generated by using a reflector antenna with different dimensions in the two principal planes(sometimes referred to as "cut" parabolas). The tilt is produced by rotating the reflector about its generator axis.

From Figure 5, at a typical intersection point, Q, we have

$$\underline{R} = \underline{S} + \underline{T}, \tag{53}$$

and because $\underline{R}$ lies on a sphere of unit radius,

$$\underline{R} \cdot \underline{R} = |\underline{R}|^2 = 1 = T^2 + S^2 + 2\underline{T} \cdot \underline{S}. \tag{54}$$

Substituting Equations (38) and (53) into Equation (54) gives the following quadratic equation in "t" for the intersection points of the antenna beam and the earth; i.e.,

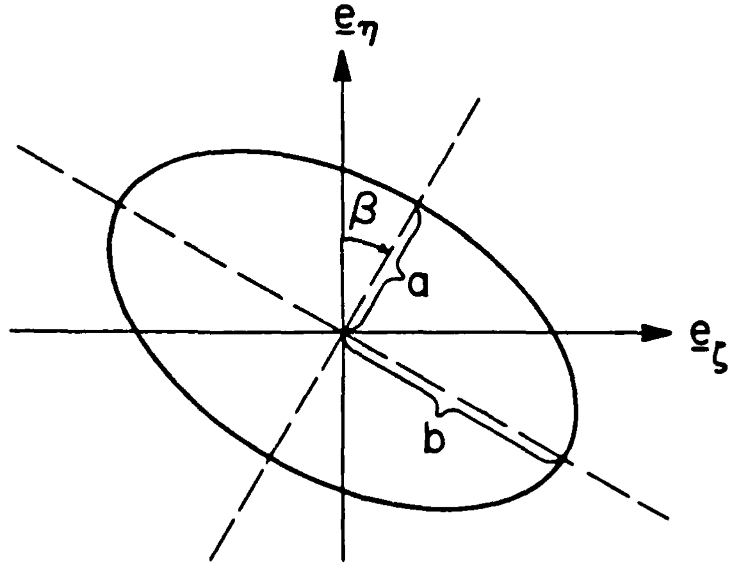

Figure 6. Cross section through titled elliptical antenna pattern.

$$a_0 t^2 + a_1 t + a_2 = 0 \tag{55}$$

where

$$a_0 = 1 + (a \cos\omega)^2 + (b \sin\omega)^2 \tag{56a}$$

$$\begin{aligned} a_1 = 2s\{&\cos\lambda_o[(F_1/d_1) - (F_2F_5)/d_1d_2) + (F_3F_6/d_2)] \\ &+ \sin\lambda_o[F_3/d_1) - (F_4F_5/d_1d_2) - (F_1F_6/d_2)]\} \end{aligned} \tag{56b}$$

$$a_2 = s^2 - 1 \tag{56c}$$

$$F_1 = \cos\lambda \cos\phi - s \cos\lambda_o \tag{56d}$$

$$F_2 = \sin\phi \; F_1 \tag{56e}$$

$$F_3 = \sin\lambda \cos\phi - s \sin\lambda_o \tag{56f}$$

$$F_4 = \sin\phi \; F_3 \tag{56g}$$

$$F_5 = a \cos\omega \cos\beta - b \sin\omega \sin\beta \tag{56h}$$

$$F_6 = a \cos\omega \sin\beta + b \sin\omega \cos\beta \tag{56i}$$

$$F_7 = a_1^2 - 4a_0a_2. \tag{56j}$$

In the computer program in Appendix A (FOOT2), the parameter F_7 is tested as ω varies from 0 to 2π to see if F_7 is less than zero, since these values of ω correspond to antenna rays which do not intersect the earth. When F_7 equals zero, the antenna rays graze the earth.

The solution of the quadratic equation (55) has two roots corresponding to an intersection point on the near or lit side of the sphere and an

intersection point on the back side or shaded side of the sphere. Since $a_1<0$, the smaller of the two roots or the intersection point on the lit side is

$$t = \frac{-a_1 - \sqrt{F_7}}{2a_0}, \tag{57}$$

and when $F_7 = 0$ or

$$a_1 = -\sqrt{a_0 a_2}, \tag{58}$$

the generators of the antenna beam are parallel to the sphere. Equation (58) defines the optical horizon or "limb line" to which we return shortly. As an example of the solution of Equation (58), suppose $\phi = 0$, $\lambda - \lambda_o = 0$; i.e., the aim point is at the intersection of the Greenwich meridian and the equator. The satellite is also at the Greenwich meridian. Also, assume the antenna beam is circular (N.B., in this case the tilt angle, β, is superfluous). Then

$$F_1 = 1 - s,$$

$$F_2 = F_3 = F_4 = 0$$

$$a_0 = 1 + a^2,$$

$$a_1 = -2s,$$

$$a_2 = s^2 - 1.$$

Then Equation (58) gives

$$t = \frac{s - \sqrt{1 - a^2(s^2 - 1)}}{1 + a^2}, \tag{59}$$

and for the antenna beam to just graze the sphere,

$$a = \frac{1}{\sqrt{s^2 - 1}} . \tag{60}$$

Substituting (59) and (60) into (52) gives

$$\underline{T} = \frac{s^2 - 1}{s} \left[\underline{e}_\xi + \frac{1}{\sqrt{s^2 - 1}} \left(\underline{e}_\eta \cos\omega + \underline{e}_\zeta \sin\omega \right) \right], \tag{61}$$

the generator of the limb line or optical horizon. Figure 7 shows the distances comprising the grazing ray in Equation (62). Note that Equation (61) gives

$$|\underline{T}| = \sqrt{\underline{T} \cdot \underline{T}} = \sqrt{s^2 - 1}, \tag{62}$$

which agrees with Figure 7.

The latitude and longitude for an arbitrary point on the limb line for an arbitrary satellite longitude, λ_o, are

$$\phi = \sin^{-1}\left(\sqrt{s^2 - 1}\, \cos\omega / s\right), \tag{63}$$

and

$$\lambda = \tan^{-1} \left\{ \frac{s \sin\lambda_o + \frac{\sqrt{s^2 - 1}}{s} \left(-\sqrt{s^2 - 1}\, \sin\lambda_o + \sin\omega\cos\lambda_o \right)}{s \cos\lambda_o - \frac{\sqrt{s^2 - 1}}{s} \left(\sqrt{s^2 - 1}\, \cos\lambda_o + \sin\omega\sin\lambda_o \right)} \right\}, \tag{64}$$

respectively.

Equation (52) represents the generator of the footprint, and one can now calculate the latitude and longitude of a typical intersection point on the footpoint. From Figure 5, we see that the latitude of a typical intersection point, ϕ_i, is determined by

$$\sin\phi_i = \underline{e}_z \cdot \underline{R}, \tag{65}$$

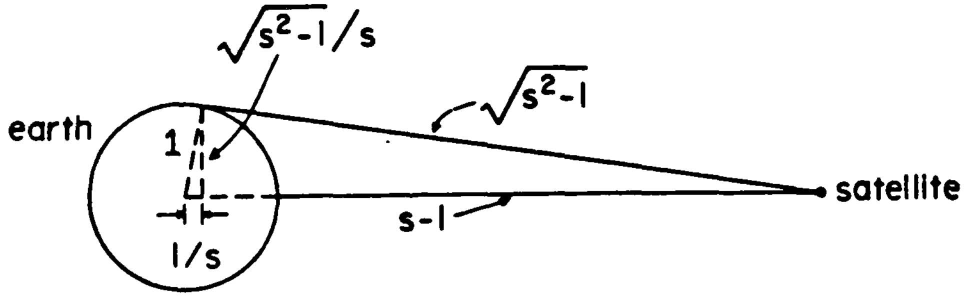

Figure 7. Geometry for derivation of limb line.

(N.B., $\underline{r}$ is a unit vector). Substituting Equations (38), (52), and (53) into (65) gives

$$\phi_i = \sin^{-1}[(t(\omega)/d_1)(\sin\phi + F_5 d_2)]. \tag{66}$$

In order to determine the proper quadrant for the longitude of a typical intersection point, λ_i, one must compute

$$F_8 = \underline{e}_y \cdot \underline{R} = \cos\phi \sin\lambda_i \tag{67}$$

and

$$F_9 = \underline{e}_x \cdot \underline{R} = \cos\phi_i \cos\lambda_i \tag{68}$$

Substituting Equations (37), (52), and (53) into (68) and (69) gives

$$F_8 = s \sin\lambda_o + t(\omega)[(F_3/d_1) - (F_4 F_5/d_1 d_2) - (F_1 F_6/d_2)] \tag{69}$$

and

$$F_9 = s \cos\lambda_o + t(\omega)[(F_1/d_1) - (F_2 F_5/d_1 d_2) + (F_3 F_6/d_2)], \tag{70}$$

so that the longitude is

$$\lambda_i = \tan^{-1}(F_8/F_9). \tag{71}$$

In Appendix A the computer program is given for computing the latitude and longitudes (ϕ_i, λ_i) of the locus of intersection points at half-degree increments in the parameter ω.

C. METHOD FOR DETERMINING WHETHER ARBITRARY POINT ON SURFACE OF A SPHERE IS INSIDE OR OUTSIDE A GIVEN FOOTPRINT

The arbitrary point may represent the location of a receiving antenna on the surface of the earth and be specified in terms of its latitude and longitude. Using the "winding number" concept (Ahlfors, 1966), we have

$$\sum_i \left\{\arg\left[f((i+1)\Delta\omega) - f_j\right] - \arg\left[f(i\Delta\omega) - f_j\right]\right\} = \begin{cases} 2\pi, & f_j \text{ inside} \\ 0, & f_j \text{ outside} \end{cases} \tag{72}$$

where $\Delta\omega = \pi/360$ and arg is the argument of the difference of the complex functions inside the brackets and

$$f(\omega) = \lambda(\omega) + i\phi(\omega) \tag{73}$$

where λ and ϕ are the longitude and latitude computed from Equations (71) and (66).

D. PROJECTIONS

The continental borders and antenna footprints are plotted according to the type of projection. That is, points on the earth defined by latitude and longitude are transformed to points in the plane of projection (the U, V plane) in which the map and footprints are plotted. The particular algorithm used to plot the world map backgrounds and overlay data, such as a satellite ground trace or antenna footprint, on the requested map projection is WORLD.

1. AZIMUTHAL PROJECTIONS

The U,V plane is tangential to the earth at the point ϕ (POLAT, POLONG), which transforms to the origin, $\phi'(0,0)$, in the U,V plane in Figure 8. Let P be a point on the earth at an angular distance, A, from the point ϕ (POLAT, POLONG). Let B be the angle between the great circle ΦP and the meridian at Φ. Then from Figure 8

$$\underline{e}_{R_1} = \underline{e}_x \cos\lambda_o + \underline{e}_y \sin\lambda_o \tag{74}$$

and

$$\underline{e}_{R_2} = \underline{e}_x \cos\delta\cos\lambda + \underline{e}_y \cos\delta\sin\lambda + \underline{e}_z \sin\delta \tag{75}$$

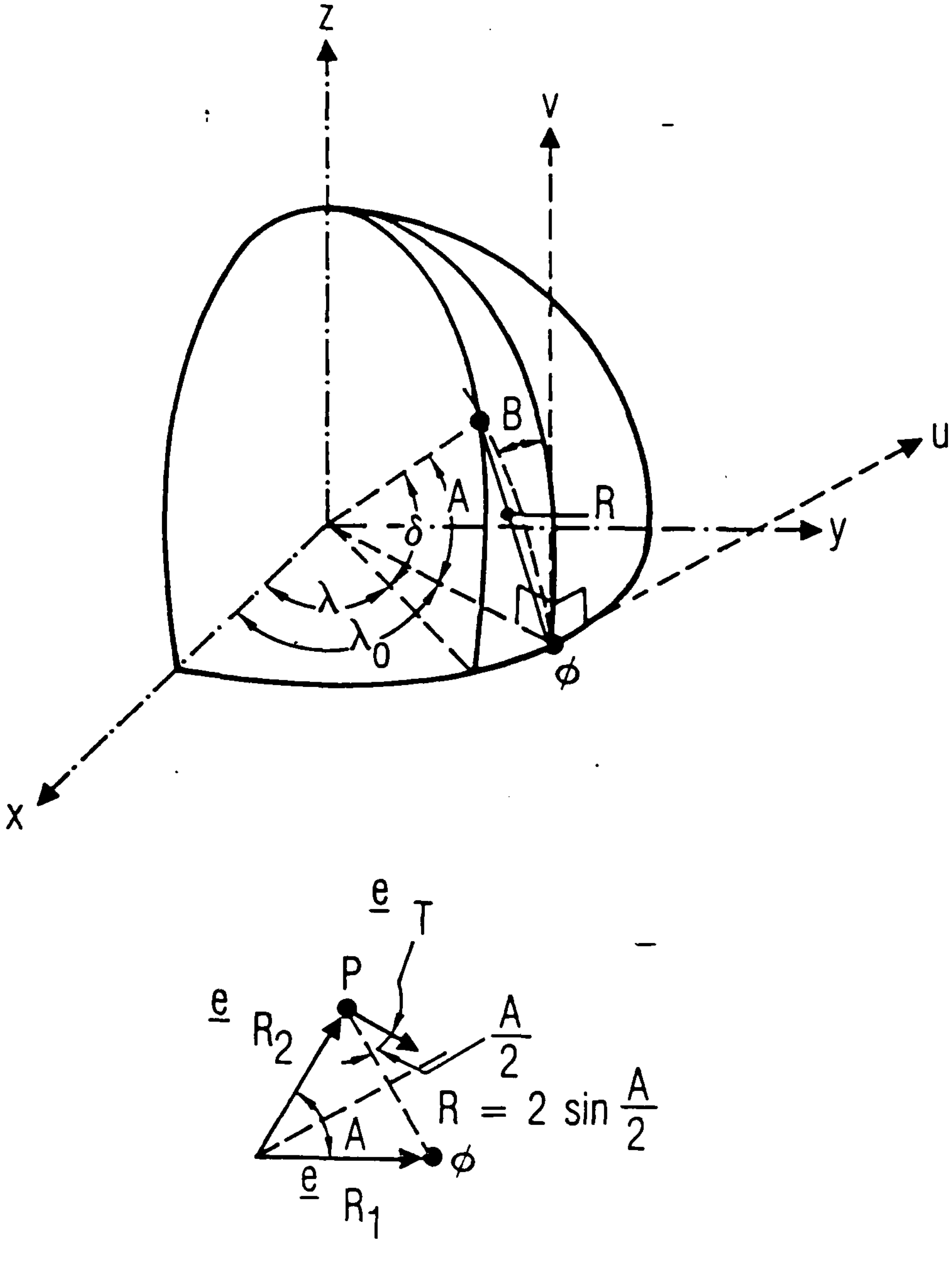

Figure 8. Transformation from U-V plane to latitude-longitude on earth.

so that

$$\underline{T} = \frac{\underline{e}_{R_2} - \underline{e}_{R_1}}{|\underline{e}_{R_2} - \underline{e}_{R_1}|} \frac{1}{\cos\frac{A}{2}} \tag{76}$$

with

$$|\underline{e}_{R_1} - \underline{e}_{R_2}| = 2 \sin \frac{A}{2} \tag{77}$$

Substituting (77) into (76) gives

$$\underline{T} = \frac{\underline{e}_{R_2} - \underline{e}_{R_1}}{\sin A} \tag{78}$$

Then

$$\cos B = \underline{e}_T \quad \underline{e}_z = \sin\delta/\sin A \tag{79}$$

and

$$\cos A = \cos\delta\cos(\lambda-\lambda_o) \tag{80}$$

with

$$R = r \sin \frac{A}{2} \tag{81}$$

Stereographic projection

R = TAN(A/2) = (1.-COSA)/SINA As A → 180°, R → ∞. Thus, the entire surface of the globe transforms to the entire U,V plane. In practice, distortion becomes great beyond R=2, or A ~ 130°.

Orthographic projection

R = SINA. This projection plots a hemisphere within radius R=1. The maximum possible value of A=90°.

Lambert equal area projection

R is calculated by the two Fortran statements

R = (1.+COSA)/SINA

R = 2./SQRT(1. + R*R).

As $A \rightarrow 180°$, $R \rightarrow 2$, and the entire surface of the globe is plotted within a radius R-2. The maximum value of A-180°.

Gnomonic projection

R = SINA/COSA As $A \rightarrow 90°$, $R \rightarrow \infty$.. A hemisphere is plotted over the entire U,V. In practice, distortion becomes great beyond R=2, or $A \sim 65°$.

Azimuthal equidistant projections

R = A (in radians) = ACOS(COSA) As $A \rightarrow 180°$, $R \rightarrow \pi$.. The entire globe surface is plotted within a radius $R = \pi$.

Cylindrical projections

The U,V plane must be imagined to be wrapped around the globe to form a cylinder, the U-axis touching the globe on some great circle (see Figure 9). The axis of the projection is perpendicular to this great circle and parallel to the V-axis. The point ϕ(POLAT,POLONG) transforms to the origin, $\phi'(0,0)$, of the projection lines on the great circle. The limits of the U-axis are defined by a cut in the cylinder parallel to its axis and diametrically opposite to ϕ. The pole of the projection, Q, is the point 90° from the great circle in the direction of +V. ROT is the angle between the V-axis and north at ϕ. These points and N, the north pole, are shown in Figure 9. Points on the surface of the globe are transformed to points in the U,V cylinder by the rule appropriate to the projection.

The latitude and longitude of Q are calculated in terms of the latitude and longitude of ϕ and the angle RϕT. The angle RϕT1 (see Figure 9) is also computed.

In Figure 9, P is some general point on the surface of the globe. A is the angular distance of P from Q. B is the angle between the great circles QN and QP. The quantities

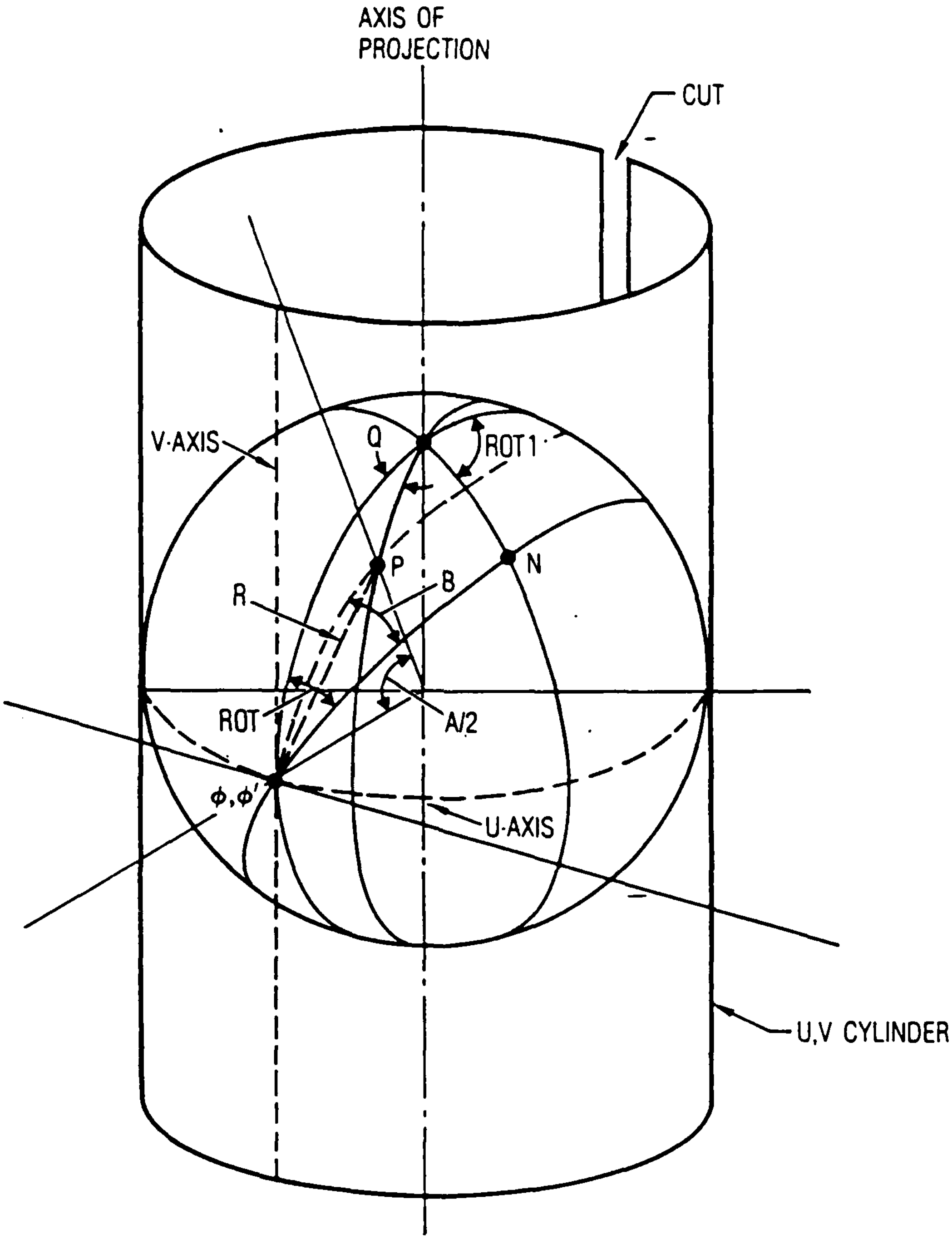

Figure 9. Definition of various parameters used in cylindrical projections.

SINA = SIN(A),   COSA = COS(A)

SINB = SINB(B),   COSB = COS(B)

SINR = SIN(ROT1),   COSR = COS(ROT1)

are computed.

Let P'(U,V) be the point on the U,V plane corresponding to the point P on the surface of the globe. Then U is proportional to the angle α (see Figure 9). The value of V depends upon the type of projection. The coordinates of P' are given below.

<u>Cylindrical equidistant projection</u>   U = α (in degrees)

= ATAN2(SIN(B+ROT1), - COS(B+ROT1))/F

= ATAN2(SINB*COSR+COSB*SINR,SINB*SINB*SINR-COSB*COSR)/F

Here division by F converts radians to degrees.

V = 90.-A (in degrees)

= 90.-ACOS(COSA)/F

The entire surface of the globe is transformed to a rectangle in the U,V plane

$-180. \leq 180.$

$-90. \leq 90.$

<u>Mercator projection with arbitrary pole</u>

U = α (in radians)

= ATAN2(SINB*COST+COSB*SINR,SINB*SINR-COSB*COSR)

V = ALOG(COT(A/2))

= ALOG((1.+COSA)/SINA)

The entire surface of the globe is transformed to an infinite rectangle in the U,V plane. As $A \to 0$, $V \to \infty$; as $A \to 180°$, $V \to -\infty$. When $\alpha = 180°$, $U = \pi$, when $\alpha = -180°$, $U = \pi$, when $\alpha = -180°$, $U = \pi$. Hence

$-\infty \leq V \leq \infty$

$-\pi \leq U \leq \pi$

In practice distortion becomes great for A < 5° or >175°.

Mollweide-type projection

The projection used is not a true Mollweide. The coordinates of P' are given by

V = COSA

U is given by the two Fortran statements

```
U = ATAN2(SINB*COSR+COSB*SINR,SINB*SINR-COSB*COSR)
    U + U*.5*SQRT(1-V*V)/ATAN(1.0)
```

The entire surface of the globe transforms to an ellipse in the U,V plane. The major and minor axes of the ellipse are along the U and V axes, respectively.

$-2. \leq U \leq 2.$

$-1.\ \underline{8}\ V \leq 1.$

III. EXAMPLES

The following examples will serve to demonstrate the use of the computer programs given in Appendix A and provide insight into the shape of footprints produced by various beams on satellites in sunsynchronous or geostationary orbit. The world map was produced using subroutine WORLDS (Gurlitz, 1981).

Consider the following example for a sunsynchronous satellite located a distance s = 1.131463449 e.r. and at an inclination of 98.1° (this corresponds to a DMSP application). The half-power beamwidth is 0.005069 rad. and $\sigma = \pi/4$. Figure 10 shows the footprints at five instants of time separated 125 millisec (the entire scan in this example takes 1.9 sec.) and for eight passes. Note how the given circular antenna beam becomes an elliptical footprint with major axis nearly parallel to a meridian at t = 0 while the footprint becomes an ellipse with major axis nearly parallel to the equator 540 millsec. later.

Figure 11 shows the received voltage versus time for the first seven passes of the sunsynchronous satellite. The period for a pass is 1900 millisec. The flattening at the top of the beam in Figure 11 results from the limited sampling of the antenna pattern data near the beam peak. The location of the transmitter and 2 and 10 dB footprints for the first three passes and part of pass 4 are shown in Figure 12. The data for the satellite antenna beamwidth were taken from the LFMR Midterm Status Review, 1 February 1985.

Next, consider a satellite in geostationary orbit at 6.619 e.r. and λ_o = 100°W. The 3-dB beamwidths in the two planes are 0.017454 rad. and 0.03491 rad. respectively and the tilt angle β = 35°. Figure 13 shows the footprint.

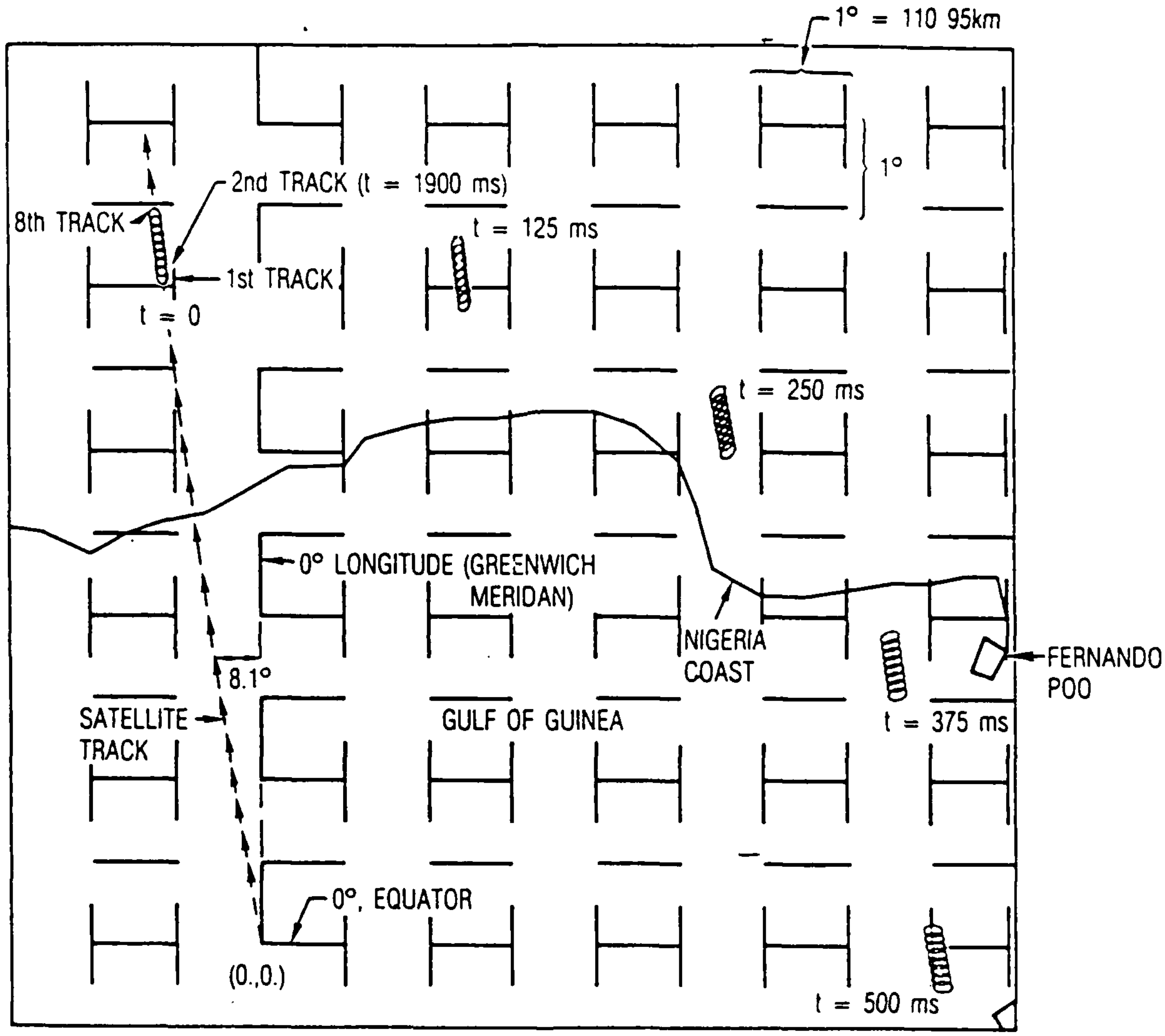

Figure 10. Footprints for satelite in sun-synchronous orbit, at an inclination of 98.1°. The separation in time of footprints in 125 millisec. The second track occurs 1900 millisec after track number 1.

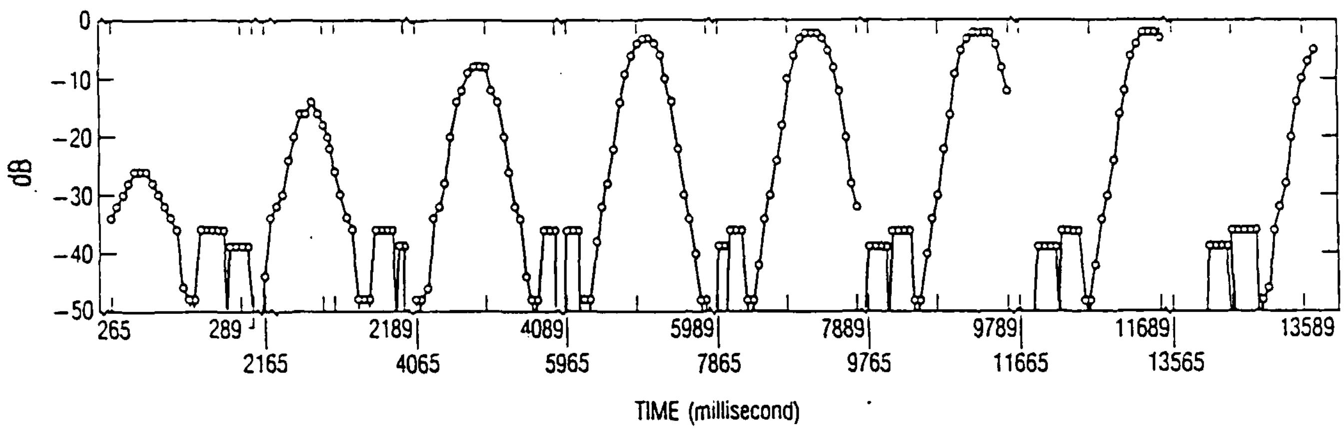

Figure 11. Time series for sun-synchrono·.. orbit at an inclination of 98.1°. The ordinate represents the received signal power from a transmitter or the ground as shown in Figure 12.

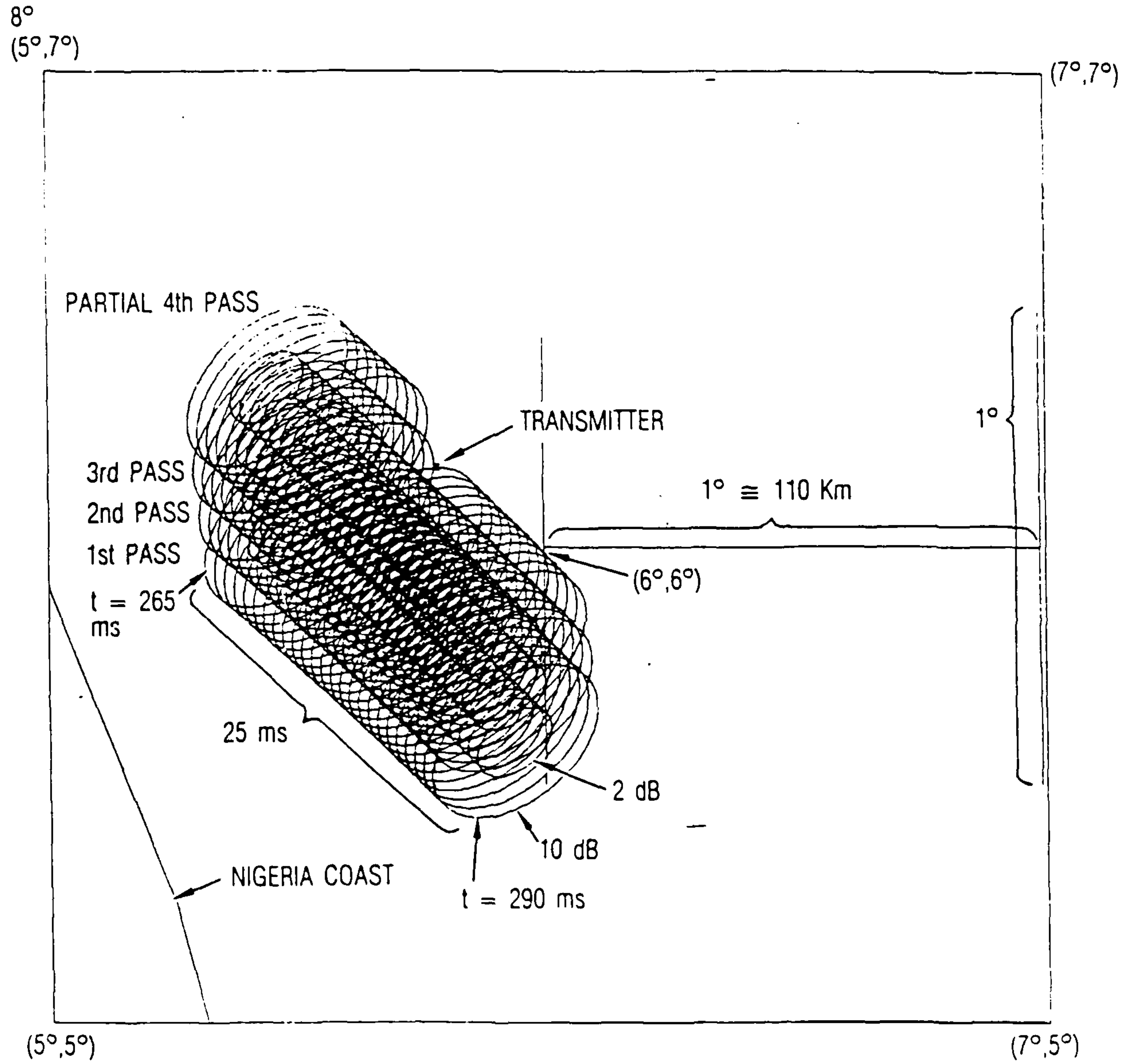

Figure 12. 2 and 10-dB footprints for 4 passes of sun-synchronous satellite near transmitter.

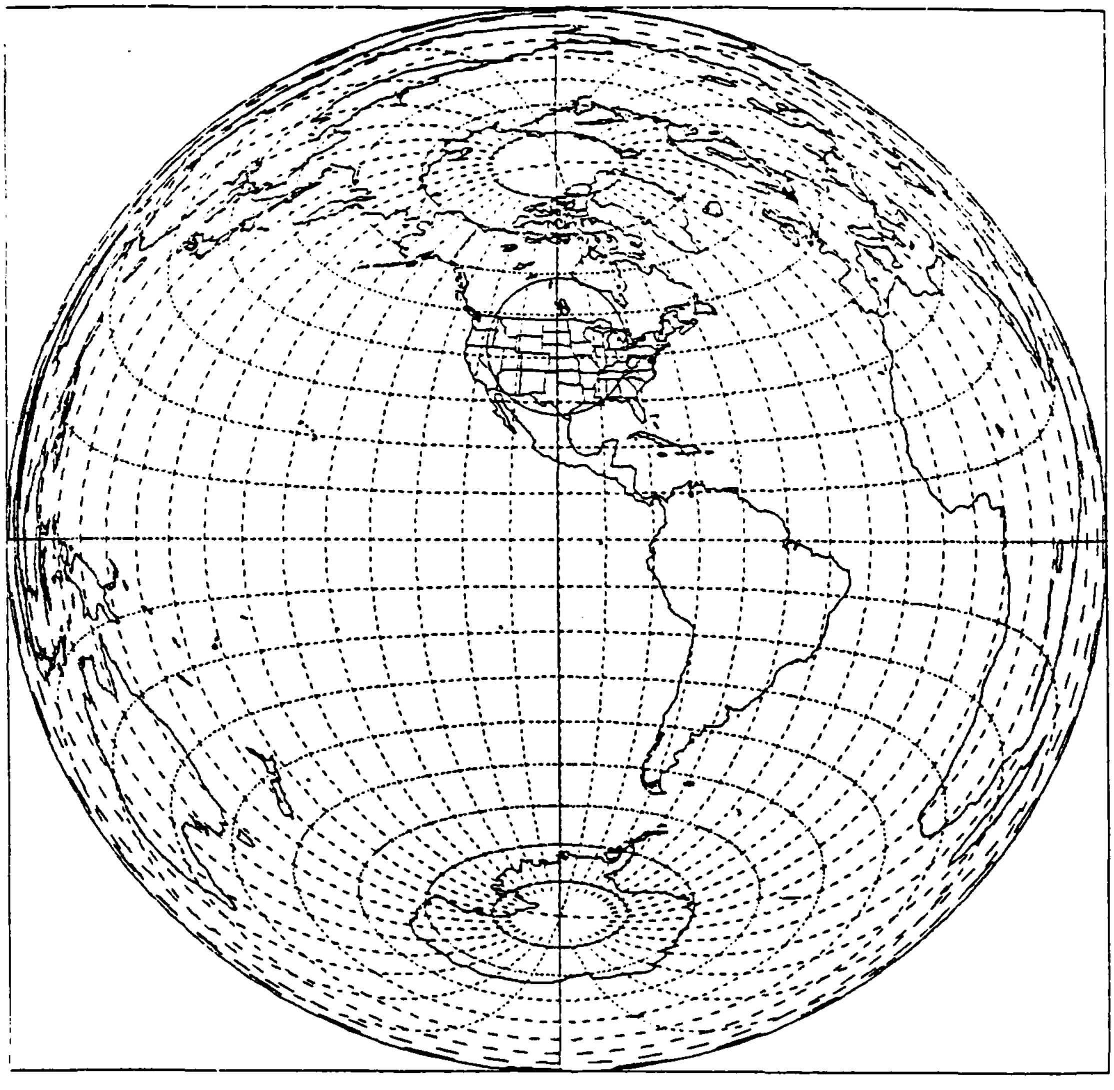

Figure 13. Footprints for a satellite in geostationary orbit. Satellite longitude is 100° West.

IV. CONCLUDING REMARKS

The vector analysis plus numerical method for generating earth coverage footprints for subsynchronous and geostationary satellites is given. The computer algorithms should provide a useful tool for determining the coverage of regions on the earth by various types of antenna beams and configurations of antennas.

These earth coverage footprints are used in the study of the requirements of the spacecraft antennas. For example, some of the considerations are: 1) a narrower spot beam on the earth requires a larger satellite antenna reflector; 2) the overlap of spot beams may result in the need for multiple frequencies or the use of orthogonal polarizations; 3) larger spacecraft antennas mean an increase in gain which in turn may result in a decrease in transmitter power; 4) the beam coverage influences the number of transponders used at the satellite; and 5) communication and radar systems and better ground resolution for radiometric remote sensing applications.

The question of satellite coverage on the surface of the earth is not new (Siocos, 1973) and the derivations of the geometrical distances are probably numerous. However, the derivations given in this report based upon vector analysis are simpler than those based upon spherical trignometry (Jacobs and Stacey, 1971; Adamy, 1974). Also, the versatility of the shape and tilt of the satellite antenna beam (i.e., elliptic in cross section with arbitrary orientation of semi-minor and semi-major axes) is an added feature not found elsewhere as far as is known. Jacobs and Stacy (1971) and Adamy (1974) develop mathematical expressions for computing earth footprints assuming circular symmetrical antenna beams.

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APPENDIX A

COMPUTER PROGRAMS

The following is a listing of the control "cards" for compiling, linking and executing the program for drawing footprints for a sun-synchronous satellite. The job is submitted as a batch job to the CDC 176 (Mainframe X).

100	F19EZ,STMFX(mainframe),P3000(priority),T5000(time limit in octal sec.), MS160000(storage),NTO(no magnetic tapes)	
110	ACCOUNT (OTT, R. 17533 F19EZ574910 A21259 54115 6530)	
120	XMIT,OUTPUT. (send output file to IBM printer)	
130	FILE,FOOT, TR=Z,BT=C,FL=80.	
140	ATTACH,FOOT,ID=17533,ST=PF6 (attach FORTRAN prog)	
150	COPY,FOOT,IN.	FTN5 (FORTRAN77) does not interpret Record type = Z format. Therefore, need the copy command which converts FOOT with record type Z to record type W (word control). The default record type on upper Cyber is W.
160	REWIND, IN.	
170	FILE,DATA,RT=Z,BT=C,FL=80.	Attach data file giving latitude and longitude of central point of projection, projection type, flag for plotting data from Tape 12, latitude and longitude grid spacings, etc.
180	ATTACH,DATA,DATA,ID=17533,ST=PF6.	
190	FTN5,I=IN,LO=R/A/M/S,PL=15000.	
200	ATTACH,WRLLIB,WORLD5LIB,ID=12730.	Attach map plotting routine
210	ATTACH,PLIB,FTN5PLOTLIB.	:Attach system plotting library
220	ATTACH,TAPE8,1BKGD,ID=12730	Attach continential background data file
230	LIBRARY,WRLLIB,PLIB.	Link
240	LOAD(LGO)	LOAD sequence, here the load consists of modules from three different files and execute as a single program
250	EXECUTE(,DATA)	
260	HARDCPY.	Command for making hardcopy plots.

The following is a FORTRAN77 program for drawing footprints from a satellite in sun-synchronous orbit together with continental boundaries.

list

```
      PROGRAM FOOT4(INPUT,TAPE5=INPUT,OUTPUT,TAPE6=OUTPUT)
      DIMENSION UFOOT(181,84),VFOOT(181,84)
      DIMENSION A(30),ID(7)
      COMPLEX W(181),Z0
      NAMELIST/DATA/IPROJ,POLON,IBEAM,TITLE,LIMIT,LATMX,
     1 LATMN,LONMN,LONMX,POLAT,SCLLAT,SCLLON
C
C          DECLARE TAPE FOR CONTINENTAL BACKGROUND DATA FILE
C
      OPEN(UNIT=8,FORM='UNFORMATTED')
      CALL PRPLOT(3H300,4HPLOT)
      ID(1) = 1
      ID(2) = 2
      ID(3) = -1
      ID(4) = 0
      ID(5) = 1
      ID(6) = 2
      ID(7) = -1
      Z0 = (0.1009,0.1076)
      WRITE(6,1) Z0
    1 FORMAT(2X,'LAT-LONG(RAD) =  ',2E20.8)
      PI = 3.1415926536
      S = 1.131463449
      AI = 98.1
      LTYPE1 = 2
      IBMIN1 = 21
      ISYMB1 = 7
      IFREQ1 = 1
      Z1 = (180./PI)*AIMAG(Z0)
      Z2 = (180./PI)*REAL(Z0)
      WRITE(12) LTYPE1,IBMIN1,ISYMB1,IFREQ1,1,Z1,Z2
      A(1) = 0.0047124
      A(2) = 0.006109
      A(3) = 0.006807
      A(4) = 0.007697
      A(5) = 0.008378
      A(6) = 0.008901
      A(7) = 0.009599
      A(8) = 0.010123
      A(9) = 0.010472
      A(10) = 0.011519
      A(11) = 0.012217
      A(12) = 0.012915
      A(13) = 0.013614
      A(14) = 0.014137
      A(15) = 0.014835
      A(16) = 0.015359
      A(17) = 0.015708
      A(18) = 0.016581
      A(19) = 0.017453
      A(20) = 0.018850
      A(21) = 0.0202458
      A(22) = 0.02094395
      A(23) = 0.021293017
      A(24) = 0.021642
      A(25) = 0.0219911
      A(26) = 0.0223402
      A(27) = 0.022689
      A(28) = 0.02740
      A(29) = 0.0366519
```

```
      A(30) = 0.042425758
C     SIGMA = (PI/4.) + 0.9*0.01745329
      SIGMA = (PI/4.)
      WRITE(6,2) AI,S,A(2),SIGMA
    2 FORMAT(2X,'INCLINATION OF ORBIT(DEG) =  ',F10.3,/,2X,
     1 'SATELLITE ALTITUDE IN EARTH RADII UNITS = ',F10.3,/,2X,
     2 '3-DB BEAMWIDTH(RAD) = ',F10.6,/,2X,
     3 'ANTENNA POINTING ANGLE(RAD) =  ',F10.3)
      R0 = (PI/180.)*AI
      R1 = COS(R0)
      R2 = SIN(R0)
      R3COS(SIGMA)
      R4 = SIN(SIGMA)
      DBETA = S*R3 - SQRT(1. - (S**2)*(R4**2))
      WRITE(6,3) DBETA
    3 FORMAT(2X,'DISTANCE OF SATELLITE TO SCAN POINT =  ',F10.3)
      LTYPE = 1
      IBMIN = 17
      ISYMB = 0
      IFREQ = 1
      SCLLAT = 1.
      SCLLON = 1.
C
C          TIME LOOP
C
      DO 10 I=1,200
      II = I-1
C
C          DECLARE TAPE FOR LATITUDE AND LONGITUED POINTS
C
       OPEN(UNIT=12,FORM='UNFORMATTED')
       IF(I.GT.25) GO TO 70
       FI = II + 265.
       GO TO 80
   70  IF(I.GT.50) GO TO 71
       FI = MOD(II,25) + 2165.
       GO TO 80
   71  IF(I.GT.75) GO TO 72
       FI = MOD(II,50) + 4065.
       GO TO 80
   72  IF(I.GT.100) GO TO 73
       FI = MOD(II,75) + 5965.
       GO TO 80
   73  IF(I,GT.125) GO TO 74
       FI = MOD(II,100) + 7865.
       GO TO 80
   74  IF(I.GT.150) GO TO 75
       FI = MOD(II,125) + 9765.
       GO TO 80
   75  IF(I.GT.175) GO TO 76
       FI = MOD(II,150) + 11665.
       GO TO 80
   76  FI = MOD(II,175) + 13565.
   80  CONTINUE
       R9 = 0.000915173*0.001*FI
       R10 = COS(R9)
       R11 = SIN(R9)
       R12 = R11*R2
       R13 = ACOS(R12)
       R14 = SIN(R13)
       R15 = R11*R1/R14
       R16 = ASIN(R15)
       R17 = (PI/2.) - R13
       R18 = (180./PI)*R17
       R19 = (180./PI)*R16
       WRITE(6,4) R18,R19
```

```
    4 FORMAT(2X,'LATITUDE OF SATELLITE(DEG) = ',E20.8,/,2X,
     1 'LONGITUDE OF SATELLITE DEGREES WEST = ',E20.8)
      BETA = 0.003306940*FI
      R20 = COS(BETA)
      R21 = SIN(BETA)
C
C           BEAMWIDTH LOOP
C
      DO 40 L=1,30
      R5 = A(L)
      R6 = 1. + R5**2
      R7 = S**2 - 1.
      R8 = R6*R7
      LASTJ = 0
      JJ = 0
C
C           FOOTPRINT GENERATOR LOOP
C
      DO 20 J=1,181
      FJ = 2.*J
      OMEGA = (PI/180.)*FJ
      R22 = COS(OMEGA)
      R23 = SIN(OMEGA)
      R24 = -R3 + R5*R22*R4
      R25 = -S*R24
      R26 = R25*R25 - R8
      IF(R26.GT.0.) GO TO 30
      LASTJ = J
      GO TO 20
   30 JJ = J - LASTJ
      T = (R25 - SQRT(R26))/R6
      R27 = S*R11*R2
      R28 = -R3*R11*R2
      R29 = -R4*R21*R1
      R30 = R4*R20*R10*R2
      R31 = R11*R2/R3
      R32 = R20*R1
      R33 = R21*R2*R10
      R34 = R3*R10
      R35 = R4*R11*R20
      R36 = -R10/R3
      R37 = S*R21*R11
      R38 = R11*R1
      R39 = -R3*R11*R1
      R40 = R4*R21*R2
      R41 = R4*R20*R10*R1
      R42 = R11*R1/R3
      R43 = R20*R2
      R44 = -R21*R1*R10
      COSTHET = R27 + T*(R28 + R29 + R30 + (R5*R22*R3/R4)*(R28 +
     1 R29 + R30 + R31) - (R5*R23)*(R32 + R33))
      STHCPH = S*R10 - T*(R34 + R35 + (R5*R22*R3/R4)*(R34 + R35 +
     1 R36) - (R5*R23)*R37)
      7HSPH = R38 + T*(R39 + R40 + R41 + (R5*R22*R3/R4)*(R39 +
     1 R40 + R41 + R42) + (R5*R23)*(R43 + R44))
      R45 = ACOS(COSTHET)
      R46 = ATAN2(STHSPH,STHCPH)
      R47 = (PI/2.) - R45
      W(J) = R46 + (0.,1.)*R47
      IF(I.GT.84) GO TO 20
      UFOOT(JJ,I) = R47/(PI/180.)
      VFOOT(JJ,I) = R46/(PI/180.)
   20 CONTINUE
      IF((L.EQ.1.AND.I.LE.84).OR.(L.EQ.9.AND.I.LE.84)) GO TO 21
      GO TO 22
   21 WRITE(12) LTYPE,IBMIN,ISYMB,IFREQ,
```

```
     1 JJ,(UFOOT(K,I),VFOOT(K,I),K=1,JJ)
  22 CONTINUE
C
C           CHECK IF ZO IS INSIDE FOOTPRINT
C
      ISUM = 0
      DO 11 K=1,181
      KK = K + 1
      IF(KK.EQ.182) KK=1
      KKK = INQUAD(W(KK) - ZO) - INQUAD(W(K) - ZO)
      JD = ID(4 + KKK)
      IF(JD.EQ.2) GO TO 12
   11 ISUM = ISUM + JD
      IF(ISUM.EQ.0) GO TO 12
      WRITE(6,13) FI,L,A(L)
   13 FORMAT(2X,'TIME(MILLISEC) = ',E20.8,2X,'3-DB BEAMWIDTH(RAD)
     1  (L = ',I5,') = ',F10.7)
      GO TO 10
   12 CONTINUE
   40 CONTINUE
   10 CONTINUE
      CLOSE(12)
      OPEN(12,FORM='UNFORMATTED')
      REWIND 12
      CALL WORLDS(5,6)
      CALL ENPLOT
      CLOSE(12)
      CLOSE(8)
      STOP
      END
      FUNCTION INQUAD(ZZ)
      COMPLEX Z,ZZ
      Z=ZZ
      IF(REAL(Z)) 1,2,3
    1 IF(AIMAG(Z)) 11,11,12
   11 INQUAD = 3
      RETURN
   12 INQUAD = 2
      RETURN
    2 IF(AIMAG(Z)) 21,22,12
   21 INQUAD = 4
      RETURN
   22 INQUAD = 0
      RETURN
    3 IF(AIMAG(Z)) 21,31,31
   31 INQUAD = 1
      RETURN
      END
```

The following is a FORTRAN77 program for drawing footprints from a satellite in geostationary orbit together with continental boundaries.

list

```
      PROGRAM FOOT2(INPUT,TAPE5=INPUT,OUTPUT,TAPE6=OUTPUT)
      DIMENSION UFOOT(361),VFOOT(361)
      NAMELIST/DATA/IPROJ,POLON,IBEAM,TITLE,LIMIT,LATMX,LATMN,
     1 LONMN,LONMX,POLAT,SCLLAT,SSCLON
      OPEN(UNIT=8,FORM='UNFORMATTED')
      CALL PRPLOT(3H300,4HPLOT)
      BEAMX = 0.03
      BEAMY = 0.04
      ROT = 0.
      PI = 3.1415926536
      S = 6.619192
      SATLON = 100.
      AIMLAT = 40.
      AIMLON = 100.
      LTYPE = 1
      IBMIN = 17
      ISYMB = 0
      IFREQ = 1
      WRITE(6,2) BEAMX,BEAMY,ROT,SATLON,AIMLAT,AIMLON
    2 FORMAT(2X,'BEAMX(RAD) = ',F10.5,2X,'BEAMY(RAD) = ',F10.5,2X,
     1 'ROTATION(RAD) = ',F10.5,/,2X,'SATLON(DEG) = ',
     2 F10.3,2X,'AIMLAT(DEG) = ',F10.3,2X,'AIMLON(DEG) = ',
     3 F10.3)
C
C           PLOT FOOTPRINT
C
      RATIO = PI/180.
      F1 = COS(AIMLON*RATIO)*COS(AIMLAT*RATIO) - S*COS(SATLON*
     1 RATIO)
      F2 = F1*SIN(AIMLAT*RATIO)
      F3 = SIN(AIMLON*RATIO)*COS(AIMLAT*RATIO) - S*SIN(SATLON*
     1 RATIO)
      F4 = F3*SIN(AIMLAT*RATIO)
      A2 = S**2 - 1.
      D1 = SQRT(1. + S**2 - 2.*S*COS(AIMLAT*RATIO)*COS(RATIO*(
     1 AIMLON - SATLON)))
      D2 = SQRT(D1**2 - SIN(AIMLAT*RATIO)**2)
      LASTJ = 0
      DO 20 J=1,361
      FJ = J
      OMEGA = FJ*RATIO
      F5 = BEAMX*COS(OMEGA)*COS(ROT) - BEAMY*SIN(OMEGA)*SIN(ROT)
      F6 = BEAMX*COS(OMEGA)*SIN(ROT) + BEAMY*SIN(OMEGA)*COS(ROT)
      A0 = 1. + (BEAMX*COS(OMEGA))**2 + (BEAMY*SIN(OMEGA))**2
      A1 = 2.*S*(COS(SATLON*RATIO)*((F1/D1) - (F2*F5/(D1*D2)) +
     1 (F3*F6/D2)) + SIN(SATLON*RATIO)*((F3/D1) - (F4*F5/(D1*D2))
     2 - (F1*F6/D2)))
      F7 = A1**2 - 4.*A0*A2
      IF(F7.GT.0.) GO TO 6
      LASTJ = J
      GO TO 20
    6 JJ = J - LASTJ
      T = ABS(-A1 - SQRT(F7))/(2.*A0)
      F10 = T/D1
      PHII = ASIN((F10)*(SIN(AIMLAT*RATIO) + F5*D2))
      F8 = S*SIN(SATLON*RATIO) + T*((F3/D1) - (F4*F5/(D1*D2))
     1 - (F1*F6/D2))
      F9 = S*COS(SATLON*RATIO) + T*((F1/D1) - (F2*F5/(D1*D2))
     1 + (F3*F6/D2))
      ALAMDI = ATAN2(F8,F9)
      UFOOT(JJ) = PHII/RATIO
```

```
      VFOOT(JJ) = ALAMDI/RATIO
   20 CONTINUE
      WRITE(12) LTYPE,IBMIN,ISYMB,IFREQ,
     1 JJ,(UFOOT(K),VFOOT(K),K=1,JJ)
      CLOSE(12)
      OPEN(12,FORM='UNFORMATTED')
      REWI12
      CALL WORLDS(5,6)
      CALL ENPLOT
      CLOSE(12)
      CLOSE(8)
      STOP
      END
```

LABORATORY OPERATIONS

The Aerospace Corporation functions as an "architect-engineer" for national security projects, specializing in advanced military space systems. Providing research support, the corporation's Laboratory Operations conducts experimental and theoretical investigations that focus on the application of scientific and technical advances to such systems. Vital to the success of these investigations is the technical staff's wide-ranging expertise and its ability to stay current with new developments. This expertise is enhanced by a research program aimed at dealing with the many problems associated with rapidly evolving space systems. Contributing their capabilities to the research effort are these individual laboratories:

Aerophysics Laboratory: Launch vehicle and reentry fluid mechanics, heat transfer and flight dynamics; chemical and electric propulsion, propellant chemistry, chemical dynamics, environmental chemistry, trace detection; spacecraft structural mechanics, contamination, thermal and structural control; high temperature thermomechanics, gas kinetics and radiation; cw and pulsed chemical and excimer laser development including chemical kinetics, spectroscopy, optical resonators, beam control, atmospheric propagation, laser effects and countermeasures.

Chemistry and Physics Laboratory: Atmospheric chemical reactions, atmospheric optics, light scattering, state-specific chemical reactions and radiative signatures of missile plumes, sensor out-of-field-of-view rejection, applied laser spectroscopy, laser chemistry, laser optoelectronics, solar cell physics, battery electrochemistry, space vacuum and radiation effects on materials, lubrication and surface phenomena, thermionic emission, photosensitive materials and detectors, atomic frequency standards, and environmental chemistry.

Computer Science Laboratory: Program verification, program translation, performance-sensitive system design, distributed architectures for spaceborne computers, fault-tolerant computer systems, artificial intelligence, microelectronics applications, communication protocols, and computer security.

Electronics Research Laboratory: Microelectronics, solid-state device physics, compound semiconductors, radiation hardening; electro-optics, quantum electronics, solid-state lasers, optical propagation and communications; microwave semiconductor devices, microwave/millimeter wave measurements, diagnostics and radiometry, microwave/millimeter wave thermionic devices; atomic time and frequency standards; antennas, rf systems, electromagnetic propagation phenomena, space communication systems.

Materials Sciences Laboratory: Development of new materials: metals, alloys, ceramics, polymers and their composites, and new forms of carbon; nondestructive evaluation, component failure analysis and reliability; fracture mechanics and stress corrosion; analysis and evaluation of materials at cryogenic and elevated temperatures as well as in space and enemy-induced environments.

Space Sciences Laboratory: Magnetospheric, auroral and cosmic ray physics, wave-particle interactions, magnetospheric plasma waves; atmospheric and ionospheric physics, density and composition of the upper atmosphere, remote sensing using atmospheric radiation; solar physics, infrared astronomy, infrared signature analysis; effects of solar activity, magnetic storms and nuclear explosions on the earth's atmosphere, ionosphere and magnetosphere; effects of electromagnetic and particulate radiations on space systems; space instrumentation.

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